Answers to Lesson 4 Mastery Check

4. On earth, \( w = mg \). If \( g \) is changed to \( g' \), the new weight would be \( w' = mg' \).

Therefore, \( \frac{w'}{w} = \frac{mg'}{mg} = \frac{g'}{g} \), or \( w' = \frac{g'}{g} w \).

On the moon, \( g' = \frac{1}{6}g \). Thus, \( w' = \frac{1}{6} w = \frac{5}{6} \text{lb} \). This is equivalent to 3.71 N.

On Jupiter, \( g' = 2.64g \). Using the above method, the weight is found as 58.7 N.

The mass is the same at all three locations.

The weight on the earth is 22.2N, and \( m = \frac{w}{g} = \frac{22.2 \text{ N}}{9.8 \text{ m/s}^2} = 2.27 \text{ kg} \).

12. The resultant of the two forces can be found using the Pythagorean theorem:

\[
F_x = \sqrt{(390 \text{ N})^2 + (180 \text{ N})^2} = 430 \text{ N}, \text{ and } \tan \theta = \frac{390 \text{ N}}{180 \text{ N}} = 2.166, \text{ so } \theta = 65.2^\circ.
\]

Thus, \( a = \frac{F_x}{m} = \frac{430 \text{ N}}{(270 \text{ kg})} = 1.59 \text{ m/s}^2 \text{ at } 65.2^\circ \text{ north of east} \).

18. Using figure 4-1 as a guide:

From \( \Sigma f_x = 0 \), \( T_1 \cos 30^\circ - T_2 \cos 60^\circ = 0 \), thus \( T_1 = 0.577T_2 \).

From \( \Sigma f_y = 0 \), \( T_1 \sin 30^\circ + T_2 \sin 60^\circ - 150 \text{ N} = 0 \).

Using the above, it is found that \( T_1 = 75 \text{ N} \), and \( T_2 = 130 \text{ N} \).
22. The total downward force acting on the person is \( mg - 100 = (80)(9.8) - 100 = 684 \text{ N} \). Using Newton's second law, the constant acceleration of the person is \( a_y = \frac{684 \text{ N}}{80 \text{ kg}} = 8.55 \text{ m/s}^2 \) downward. Then

\[
\frac{v_y^2}{v_{0y}^2} = 2a_y(\Delta y) = 0 + 2(8.55)(30)
\]

which returns \( v_y = 23 \text{ m/s} \).

32. In the vertical direction, \((8000 \text{ N})\sin 65^\circ - w = 0\), which simplifies to \( w = 7250 \text{ N} \).

\[
m = \frac{w}{g} = \frac{7250 \text{ N}}{9.8 \text{ m/s}^2} = 739.8 \text{ kg}.
\]

Along the horizontal, the second law becomes \((8000 \text{ N})\cos 65^\circ = (739.8 \text{ kg})a_x\), so \( a_x = 4.57 \text{ m/s}^2 \).

42a. Using the given time and distance, \( x = v_0t + \frac{1}{2}at^2 \) gives us

\[
l = 0 + \frac{1}{2}a(16 \text{ s}^2),
\]

or \( a = 0.125 \text{ m/s}^2 \). Since the acceleration of the 4 kg mass is then 0.125 m/s², Newton's second law gives \( T - 39.2 \text{ N} = (4 \text{ kg})(0.125 \text{ m/s}^2) \) or \( T = 39.7 \text{ N} \). Then, the second law for the 9 kg mass gives \( \Sigma f_y = ma_y = 0 \), or \( N = w \cos 40^\circ = (9 \text{ kg})(9.8 \text{ m/s}^2) \cos 40^\circ = 67.6 \text{ N} \), and \( \Sigma F_x = ma_x \), or \( mg \sin 40^\circ - T - f = (9 \text{ kg})a \), which yields

\[
f = (9 \text{ kg})(9.8 \text{ m/s}^2) \sin 40^\circ - 39.7 \text{ N} - (9 \text{ kg})(0.125 \text{ m/s}^2) = 15.9 \text{ N} \). Therefore,

\[
\mu = \frac{f}{N} = \frac{15.9 \text{ N}}{67.6 \text{ N}} = 0.235.
\]

42b. The acceleration was found to be 0.125 m/s² in problem 42a.

42c. The tension in the 4 kg mass was found as 39.7 N in problem 42a.

54a. At the point of pending motion, \( f_s = (f_s)_{\text{max}} = \mu_sN = \mu_smg \cos \theta \). Since equilibrium still exists, this also equals the force of gravity parallel to the incline, or \( \mu_smg \cos \theta = mg \sin \theta \). Therefore,
\[ \mu_\varepsilon = \tan \theta = \frac{h}{\sqrt{L^2 - h^2}}. \]

54b. Using \( x = L = \frac{1}{2}at^2 \) gives \( a = \frac{2L}{t^2} \).

54c. \( \sin \theta = \frac{h}{L} \), or \( \theta = \sin^{-1}(\frac{h}{L}) \)

54d. Looking at the forces that are parallel to the incline (figure 4-2), we get

\[ ma = mg \sin \theta - \mu_s \cos \theta, \text{ so that} \]
\[ a = (\sin \theta - \mu_s \cos \theta)g = (\mu_k - \mu_s)g \cos \theta. \]

It follows:

\[ \mu_k = \frac{(g \sin \theta - a)}{(g \cos \theta)} = \frac{(g \sin \theta - a)}{g \sqrt{1 - \sin^2 \theta}}. \]

Substituting for \( a \) and \( \sin \theta \) from problems 54b and 54c reduces this to

\[ \mu_k = \frac{h - \frac{2L^2}{gt^2}}{\sqrt{L^2 - h^2}}. \]

58a. (Refer to figure 4-3.)

Equilibrium indicates

(1) \(- \Sigma F_x = F \cos \theta + f_{sx} - mg \sin \theta = 0\)
(2) \(- \Sigma F_y = N - F \sin \theta - mg \cos \theta, \text{ or} \)
\[ N = F \sin \theta + mg \cos \theta. \]

For minimum \( F \) (motion is pending),

(3) \(- f_s = (f_s)_{max} = \mu_s N \)

With \( m = 2 \text{ kg}, \theta = 60^\circ, \text{ and } \mu_s = 0.3, \) equations (2) and (3) become:

\[ N = 0.866F + 9.8 \text{ N}, \text{ and } f_s = -0.26F + 2.94 \text{ N}. \]

Therefore, equation (1) gives
\[ 0.5F + 0.26F + 2.94\text{ N} - 16.97\text{ N} = 0 \text{ or } F = 18.5\text{ N}. \]

58b. From equation (2) in problem 58a, \( N = 25.8\text{ N}. \)

58c. If \( \theta = 30^\circ \), the method used in problem 58a gives \( F = 4.63\text{ N}. \)

62. (Refer to figure 4-4.)

The resultant x component of the two forces exerted on the traffic light by the cables is 0, since \( (60\text{ N})\cos 45^\circ - (60\text{ N})\cos 45^\circ = 0 \). Thus, the resultant y component is 84.9 N vertically upward. The forces on the traffic light are the 84.9 vertical force exerted by the cables and the weight. The resultant of these two must equal zero, from the first law. Thus, \( w = 84.9\text{ N}. \)

74a. The friction force between the box and the truck bed causes the box to move with the truck.

74b. The maximum value of the acceleration the truck can have before the box slides can be found by finding the maximum value of the static friction force on the box. This is \( f_{\text{max}} = \mu_s N = \mu_s mg \). Thus, from Newton's second law,

\[
a_{\text{max}} = \frac{f_{\text{max}}}{m} = \mu_s g = 0.3(9.8 \text{ m/s}^2) = 2.9 \text{ m/s}^2.
\]