n lesson 1 we mentioned that an important part of mathematics is recognizing and describing patterns. We use inductive reasoning to recognize patterns and reach logical conclusions. One may use deductive reasoning to prove the validity of these conclusions, thus creating properties or theorems. In this lesson we will discuss the difference between inductive and deductive reasoning, and then demonstrate how to use deductive reasoning to write geometric proofs that verify the truth of accepted geometric theorems.

### What You’ll Learn to Do

1. Recognize the difference between conclusions reached by inductive reasoning and those reached by deductive reasoning.
2. Write the inverse, converse, and contrapositive of a statement.
3. Determine the truth of statements demonstrating geometric properties.
4. Learn the postulates of Euclidean geometry and how they relate to deductive reasoning.
5. Complete geometric proofs by supplying logical statements or reasons that lead to a desired conclusion.

### Objective 1

Recognize the difference between conclusions reached by inductive reasoning and those reached by deductive reasoning.

### Inductive Reasoning

Inductive reasoning is the process of observing, recognizing, and describing patterns, and then making predictions or generalizations from these patterns. In science, predictions and generalizations from patterns are called hypotheses; in math we call them conjectures. Inductive reasoning is used in many common experiences of our lives. For example, I have noticed that the mail carrier usually delivers our mail between 11 a.m. and 12:30 p.m. So, I don’t even bother to check the mail until 1 p.m. Consider the inductive reasoning: I observe when the mail carrier comes. I recognize and describe the pattern by noticing that she comes between 11 a.m. and 12:30 p.m. I generalize this pattern by deciding to check the mail at 1 p.m.

Many of us use inductive reasoning when we decide what time to get up in the morning. I have to be at school by 7:20 a.m. I have noticed that it usually takes me 45 minutes from the time I get up, to leave the house, a pattern that I have recognized and just described. I only need 5 minutes to go from my home to the school, another pattern. I have also found that I hit the snooze button on my alarm for about half an hour before I get out of bed, a third pattern. Putting all three patterns together, I conclude that I need to set my alarm for 6:00 a.m., the generalization.
Inductive reasoning leads to very good predictions and generalizations, but these predictions and generalizations do not always have to come true. The mailman does not always deliver my mail before 1 p.m. Some mornings I get to school early, and once in a while I’m late. This means that when we use inductive reasoning to form conjectures, we are simply stating what we believe to be true about the patterns we recognize. For example, early mathematicians used to believe that every number is rational, or could be written as a fraction of two integers. Eventually, they realized that numbers like  and  could only be approximated by fractions, and they discovered irrational numbers.

In this lesson we will learn how to use mathematical logic to prove that some conjectures are always true. This cannot be done with inductive reasoning, but don’t underestimate the importance of inductive reasoning. All discoveries begin by someone recognizing and generalizing some pattern. The following examples demonstrate how to create geometric conjectures.

**Example 1**
What conjecture can we form about two angles that are congruent and complements?

**Procedure**
Since complements add up to 90°, let’s begin by constructing a right angle. To do this we may use any of the three methods described in examples 6, 19, and 20 in lesson 1. (I will use the method of example 20.) After constructing a right angle, bisect it to form two congruent angles that are complements. See Figure 2–1.

![Figure 2–1](image)

In Figure 2–1  is a right angle, and . Hence,  and  are congruent complements.

Now use a protractor to measure  and . I found that  and . This make sense because we know the angles are congruent and add up to 90°. So, let’s describe what we have found.

**Conjecture**
Congruent complements measure 45°.

We cannot say that this conjecture is absolutely true because our construction or measurement could be inaccurate. Later on we will prove that this conjecture is true.

**Example 2**
What conjecture can you form about two angles that are congruent supplements?

Take a moment and write your own conjecture for congruent supplements. A proof of the desired conjecture is given as a Let’s Check Your Mastery exercise.

**Example 3**
What conjecture can we form for vertical angles? (Review the definition of vertical angles in lesson 1 if necessary.)

![Figure 2–2](image)

Figure 2–2 shows six pairs of vertical angles:  and ,  and ,  and , and . Measure all twelve angles with a protractor, and write a conjecture for what you observe about each pair of vertical angles. Did you come up with a conjecture similar to the conjecture below?

**Conjecture**
Vertical angles are congruent.

**Deductive Reasoning**

Deductive reasoning is the process of using facts that are accepted as true to demonstrate the truth of other statements. For example, suppose that Devin and Peter take the same test in a class. If the teacher says that Peter’s score of 95 percent is the highest score on the test, then we use deductive reasoning to conclude that Devin’s score is less than 95 percent.
Detectives use deductive reasoning to solve crimes and catch criminals. The fictional detective most famous for using deductive reasoning was Sherlock Holmes. He always solved the mystery and found the criminal by deducing information from the clues and other known facts. Dr. Watson was often amazed by Sherlock Holmes’s conclusions, but after following the deductive arguments step-by-step, he always accepted them as true. As you begin to use deductive reasoning you may feel very much like Dr. Watson. With careful study you will soon follow, understand, and accept the step-by-step arguments described. Eventually, after a lot of practice, you can become as good at deductive reasoning as Sherlock Holmes. Think about the deductive reasoning used in this next example.

**Example 4**

Suppose you see a friend outside of a movie theater. You ask why she is waiting outside. She says, “I’m waiting for my brother and his wife. Oh, there he is now.” She then waves at a man and a woman coming toward the two of you. You don’t have to ask which person is her brother. By deductive reasoning you know that her brother is the man.

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**Let’s Check Your Mastery**

Answer the following questions and then check your answers in the Answer Key at the back of this course manual. Do not submit your answers to Independent Study.

Determine if the conclusion is an example of inductive or deductive reasoning.

1. Jack goes fishing every morning during a week-long camping trip. Each morning he catches fish between 6:30 a.m. and 7:30 a.m. So, Jack concludes that the fish always eat at this time of the day.

2. Jill’s math teacher always hands back exams in the order of worst score to best score. She notices that her test is on the bottom of the stack. Hence, Jill thinks that she got an A on the exam.

3. Doug is trying to remember which opponent his football team played first this season. He remembers that the first team they played wore red uniforms. Since his cousin’s team wears green uniforms, he knows that they did not play his cousin’s team first this season.

4. Mr. Johanson notices that Allison has come to class late five times in the last two weeks. When Allison is not in her seat at the beginning of class, he expects her to come in late.

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5. At Sidney’s dance recital, her mother notices that the printed program listed Sidney’s dance number as the seventh one to be performed. After counting six dance numbers, her mother prepares to take pictures because she expects Sidney to be on stage next.

**Interactive Math Lab**

If you need help with these concepts, try problems 9–10 from lesson 2 on your Math Lab DVD.

Whether we use inductive or deductive reasoning to form our conclusions, as people we often make mistakes. Even the very best mathematicians and scientists have made mistakes. A simple way to determine that a conclusion is wrong is to find a counter example. A **counter example** is anything that fits the conditions specified in a conjecture but does not satisfy its conclusion. It only takes one counter example to show that a conjecture is false. In example 1, we formed the conjecture “congruent complements each measure 45°.” If we can find a pair of congruent complements that do not each measure 45°, then this conjecture would be false.

As you consider the five Let’s Check Your Mastery questions, you might notice that a counter example could exist for most of the conclusions. Students often think that this indicates that the person is using inductive reasoning. In two of the problems deductive reasoning was used, but the conclusion may still turn out to be incorrect. In question 2, Jill uses deductive reasoning but reaches a slightly flawed conclusion. By deductive reasoning she should only conclude that she will receive the best score on the test, but she adds the condition that the best score is an A. There is a chance that the best score is not an A. It is a typical mistake when using deductive reasoning to try to stretch the reasoning beyond what is really guaranteed. In question 5, Sidney’s mother uses deductive reasoning correctly, but Sidney may not be on stage next. If Sidney is not on stage next, it shows that the accepted fact, the printed recital program, is flawed. Regardless, the mother still used good deductive reasoning.

In mathematics it is critical that deductive reasoning always leads to the correct conclusion. Mathematicians do not allow themselves to stretch the meaning of a phrase like “best score” to mean “an A grade.” Hence, it is important to understand the precise meaning of all geometrical terms as you proceed through this course. In order to avoid flaws in the accepted facts, mathematical properties are absolute and without exception. They will be described later in objectives 3 and 4.
Later on you will be asked to prove that geometric shapes have certain properties or to use these properties to do calculations. Both will require you to use deductive reasoning. In order to use deductive reasoning effectively, you must understand the rules of logic. Logic always starts with a premise, usually a conditional statement. A conditional statement is a statement that relates two ideas with an “if-then” conjunction. Often conditional statements say, “if one thing is true, then another thing logically follows.” The conditional statement for our Devin and Peter example would be, “if Peter’s score is the highest in the class, then all other scores are less than his 95 percent.” Since Peter does have the highest score in the class, it logically follows that Devin’s score is less than Peter’s 95 percent.

For any conditional statement there are three variations of the statement: the inverse, the converse, and the contrapositive. The inverse of a conditional statement is written by negating both the “if” and “then” parts of the statement. The inverse of the statement above is, “if Peter’s score is not the highest in the class, then not all other scores are less than his 95 percent.” The converse of a conditional statement is written by reversing the “if” and “then” parts of the statement. The converse of the statement above is, “If all other scores are less than his 95 percent, then Peter’s score is not the highest score in the class.” The contrapositive of a conditional statement is written by negating and reversing the “if” and “then” parts of the statement. The contrapositive of the statement above is, “if not all other scores are less than his 95 percent, then Peter’s score is not the highest score in the class.”

**Example 5**

Write the inverse, converse, and the contrapositive of the statement, “if two segments are congruent, then they have the same measure.”

**Solution**

**Inverse:** If two segments are not congruent, then they do not have the same measure.

**Converse:** If two segments have the same measure, then they are congruent.

**Contrapositive:** If two segments do not have the same measure, then they are not congruent.

If you look at the truth of the statements we have written so far, you will notice that each variation is a true statement. This is what leads to the invalid logic that many students mistakenly use.

Many people think that if a conditional statement is true, then its inverse, converse, and contrapositive are also true. This is the case with our two examples, but our examples are special. Our examples are definitions. The definition of “highest score” is that all other scores are less than it. The definition of “congruent” is having the same measure. For any definition the inverse, converse, and contrapositive will be true. Example 6 considers a statement that is not a definition.

**Example 6**

The conditional statement used in the logic of example 4 is, “if a person is her brother, then the person is a man.” Write the inverse, converse, and contrapositive of this statement and decide if they are true or false.

**Solution**

**Inverse:** If a person is not her brother, then the person is not a man. False, there are millions of men who are not her brother.

**Converse:** If a person is a man, then the person is her brother. False, again there are millions of men who are not her brother.

**Contrapositive:** If a person is not a man, then the person is not her brother. True.

Since the inverse and converse are false, we know that the definition of “brother” is not being a man. Being a man is just a property of “brother.” Notice that the contrapositive is true. The Law of Contrapositives states that if a conditional statement is true, then its contrapositive will also be true. Consequently, when using deductive reasoning a conditional statement and its contrapositive are interchangeable, but not its inverse or converse. Hence, invalid logic occurs most often when someone assumes the inverse or converse of a statement is true just because the statement is true.
Properties

In this section we will review some mathematical properties that are accepted as true. Thus, we can use them to prove that our conjectures are true. Since we work with measurements, our first properties are the properties of equality for numerical values. The following table gives the name, a description, and an example of each property of equality.

<table>
<thead>
<tr>
<th>Name</th>
<th>Description</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reflexive</td>
<td>Any number is equal to itself.</td>
<td>$8 = 8$</td>
</tr>
<tr>
<td>Symmetric</td>
<td>Equality can be expressed in any order.</td>
<td>$x = 10$ and $10 = x$ are equivalent.</td>
</tr>
<tr>
<td>Transitive</td>
<td>Two expressions equal to the same value must be equal to each other.</td>
<td>$5 + 6 + 7 = 11 + 7$ and $11 + 7 = 18$, so $5 + 6 + 7 = 18$.</td>
</tr>
<tr>
<td>Substitution</td>
<td>If two values are equal, then they can be used interchangeably.</td>
<td>If $x = 3$, then $2x + 11 = 2(3) + 11 = 6 + 11 = 17$.</td>
</tr>
</tbody>
</table>
| Addition | Adding the same thing to both sides of an equation does not change the equality. | $a - 12 = 4$  
$(a - 12) + 12 = 4 + 12$  
$a = 16$ |
| Subtraction | Subtracting the same thing from both sides of an equation does not change the equality. | $b + 2 = 30$  
$(b + 2) - 2 = 30 - 2$  
$b = 28$ |
| Multiplication | Multiplying by the same thing, except zero, on both sides of an equation does not change the equality. | $r = 16$  
$2r = 2(16)$  
$2r = 32$ |
| Division | Dividing by the same thing, except zero, on both sides of an equation does not change the equality. | $4x = 56$  
$4x = 56$  
$4 \div 4$  
$x = 14$ |
| Distributive | $a(b + c) = ab + ac$ | $4x + 5x = (4 + 5)x = 9x$ |

These properties should all be review from algebra. Since equality and congruence are very similar, the properties of congruence for geometric objects are nearly identical to the properties of equality. This next table gives the names, a description, and an example of each property of congruence.
The main difficulty with learning these properties is to learn the difference between the transitive property and the substitution property. There is only a subtle difference between the two properties. In fact, the substitution property can be replaced by using a combination of the other properties. Hence, some geometry courses do not include the substitution property, but we will because it allows us to do problems more quickly. One way to recognize the difference between these properties is to remember that the transitive property allows you to replace everything on one side of an equation with something else to which it is equal. The substitution property allows you to replace any part of an expression by something else to which it is equal.

Table 2–2

<table>
<thead>
<tr>
<th>Name</th>
<th>Description</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reflexive</td>
<td>Any geometric object is congruent to itself.</td>
<td>$\overline{AB} \cong \overline{AB}$, $\angle PTS \cong \angle PTS$</td>
</tr>
<tr>
<td>Symmetric</td>
<td>Congruence can be expressed in any order.</td>
<td>$\overline{DE} \cong \overline{MN}$ and $\overline{MN} \cong \overline{DE}$ are equivalent.</td>
</tr>
<tr>
<td>Transitive</td>
<td>Two objects congruent to the same thing must be congruent to each other.</td>
<td>If $\angle PTS + \angle STW \cong \angle PTS$ and $\angle PTW \cong \angle ABC$, then $\angle PTS + \angle STW \cong \angle ABC$.</td>
</tr>
<tr>
<td>Substitution</td>
<td>If two figures are congruent, then they can be used interchangeably.</td>
<td>If $\overline{AB} \cong \overline{DE}$ and $\overline{DE} \cong \overline{EF}$, then $\overline{DF} \cong \overline{AB} + \overline{EF}$.</td>
</tr>
</tbody>
</table>

Let’s Check Your Mastery

Choose the property that justifies each conclusion.

8. If $\overline{DE} = \overline{DR} + \overline{RE}$ and $\overline{DR} = \overline{PQ}$, then
   - $\overline{DE} = \overline{PQ} + \overline{RE}$ because of the
     a. transitive property
     b. addition property
     c. symmetric property
     d. substitution property

9. If $\overline{DE} \cong \overline{DR} + \overline{RE}$ and $\overline{DR} + \overline{RE} \cong \overline{PQ} + \overline{RE}$, then
   - $\overline{DE} \cong \overline{PQ} + \overline{RE}$ because of the
     a. transitive property
     b. addition property
     c. symmetric property
     d. substitution property

10. If $\overline{DR} = \overline{PQ}$ and $\overline{RE} = \overline{ST}$, then
    - $\overline{DR} + \overline{RE} = \overline{PQ} + \overline{ST}$ because of the
      a. transitive property
      b. addition property
      c. symmetric property
      d. substitution property

11. If $\overline{DR} + \overline{RE} = \overline{PQ} + \overline{RE}$, then $\overline{DR} = \overline{PQ}$ because of the
    a. reflexive property
    b. addition property
    c. subtraction property
    d. substitution property

12. We know that $\overline{RE} \cong \overline{RE}$ because of the
    a. reflexive property
    b. symmetric property
    c. transitive property
    d. substitution property

13. If $\overline{DR} \cong \overline{PQ}$, then $\overline{PQ} \cong \overline{DR}$ because of the
    a. reflexive property
    b. symmetric property
    c. transitive property
    d. substitution property

Interactive Math Lab

If you need help with these concepts, try problems 1–5 from lesson 2 on your Math Lab DVD.
Objective 4
Learn the postulates of Euclidean geometry and how they relate to deductive reasoning.

Conjectures, Postulates, and Theorems

As we said in the first section of this lesson, a conjecture is something we believe to be true through the process of inductive reasoning. We have also said that we will use deductive reasoning to prove that our conjectures are true. Once a conjecture has been proven true through valid logic, we call it a theorem. To show that a conjecture is incorrect, we simply find a counter example. Once a conjecture has been shown incorrect by a counter example, it is discarded.

Example 6
Demonstrate that the conjecture, “any three points are collinear,” is incorrect by showing a counter example.

Counter example
Figure 2–3 shows three points, A, B, and C, which cannot lie on a single line.

Figure 2–3

In the Deductive Reasoning section we mentioned that deductive reasoning uses facts that are accepted as true to demonstrate the truth of other statements. This means that in order to prove that our conjectures are theorems, we must have some accepted facts. In geometry, postulates are facts that we accept as true without formally proving that they are true. Notice that theorems are proven to be true and postulates are simply accepted as true. Postulates are like the rules that everyone agrees to follow when they begin to play the “geometry game.”

In approximately 300 B.C. a Greek mathematician named Euclid wrote his book, Elements, which was the first extensive work using postulates and deductive reasoning to explain the development of geometric results. Euclid used five postulates to set up what we now call Euclidean geometry.

The following list summarizes the five postulates:

1. A line can be drawn through any two points.
2. A segment can be extended indefinitely to form a straight line.
3. A circle is determined by a center point and a fixed distance called the radius.
4. All right angles are congruent.
5. Given a line and a point not on that line, there exists exactly one line through the given point parallel to the given line. (Also known as Playfair’s Axiom.)

Euclid used these postulates, common notions that roughly describe our properties of equality and congruence, and definitions to prove the known geometric results of his time. Our purpose in this course is not to recreate Euclid’s Elements nor reprove his results. We simply want to understand the power of inductive and deductive reasoning for proving and using mathematical results. Hence, we will state many important theorems without giving their proofs. We will use these theorems to make calculations, demonstrate geometric properties, and prove other theorems.

As you study Euclid’s postulates, you may think that they are so obvious or simple that it is almost unnecessary to even mention them. Modern mathematicians have explored the effect that changing these postulates has on the properties of Euclidean geometry. Changing the fifth postulate led to two new geometries called non-Euclidean geometries. Changing the fifth postulate to read, “Any two lines always intersect,” resulted in the development of elliptic geometry. Elliptic geometry describes the properties for working on a spherical surface like a ball or a planet. Imagine wrapping two strings around a basketball so that they divide the ball into two halves. These two strings would always have to intersect. A geometry without parallel lines, is very useful for planning airplane routes around the spherical shape of our earth.

Hyperbolic geometry came as a result of changing the fifth postulate to, “Given a line and a point not on that line, there are at least two lines through the given point parallel to the given line.” In hyperbolic geometry surfaces are similar to that of a saddle or funnel. Imagine the surface of some water in a bathtub as it goes down the drain. It curves.
toward the drain. Albert Einstein used hyperbolic geometry to develop and describe his general theory of relativity.

Modern mathematicians have developed many other non-Euclidean geometries, but in this course we will only study the properties of Euclidean geometry. Don’t let the idea of other geometries confuse you. Just remember that postulates give the accepted facts of geometry and determine the properties that can be proven as theorems.

Let’s Check Your Mastery

Answer the following questions and then check your answers in the Answer Key at the back of this course manual. Do not submit your answers to Independent Study.

Determine if the following statements are true or false.

14. Conjectures can always be proven true.
   a. true
   b. false

15. Postulates can always be proven true.
   a. true
   b. false

16. A conjecture becomes a theorem after it has been proven true by valid logic.
   a. true
   b. false

17. We can use existing theorems to prove new theorems.
   a. true
   b. false

Objective 5

Complete geometric proofs by supplying logical statements or reasons that lead to a desired conclusion.

Writing Proofs

A proof is a formal argument that shows how a set of premises logically leads to a desired conclusion. There are several different ways that we can write an acceptable proof. Paragraph proofs are written just as if we were explaining our argument to another person. Flow charts show how one idea leads to another. A two-column proof lists step-by-step statements in one column and corresponding reasons that justify the statements in a second column. The key to any type of proof is that valid logic leads from one statement to the next and the appropriate accepted definition, property, postulate, or theorem is used to justify each statement.

There are three things that we can do to make a proof easier to write. First, whenever possible, draw a diagram that represents the geometric ideas described in the premises. Second, list the premises, or given information, for the proof as concisely as possible. Third, describe the desired conclusion, or what you are trying to prove. The following examples should help you better understand how to write a proof.

Example 7

Write a flow chart proof for the conjecture, “congruent complements measure 45°.”

Set Up

First draw a diagram, then list the given information and what we are trying to prove.

Figure 2–4
Example 8

Write a two-column proof for the conjecture, “congruent complements measure 45°.”

Set Up

We will use the same set up from example 7. In fact, the flow chart will help us write the two-column proof.
As you compare examples 7 and 8, you should notice that there is very little difference between the two proofs. A flow chart proof can be clearer on how one statement leads to the next, but a two-column proof usually takes less space. From now on, we will do most of our proofs in the two-column format.

**Example 9**
Prove that the complements of congruent angles are congruent. (See Figure 2–7.)

**Figure 2–7**

![Figure 2–7](image)

**Figure 2–8**

**Proof**

**Given:** \( \angle PUQ \cong \angle TUS \)

\( \angle PUQ \) and \( \angle QUR \) are complements.
\( \angle TUS \) and \( \angle SUR \) are complements.

**Statements**

1. \( \angle PUQ \) and \( \angle QUR \) are complements.
2. \( m\angle PUQ + m\angle QUR = 90^\circ \)
3. \( \angle TUS \) and \( \angle SUR \) are complements.
4. \( m\angle TUS + m\angle SUR = 90^\circ \)
5. \( m\angle PUQ + m\angle QUR = m\angle TUS + m\angle SUR \)
6. \( \angle PUQ \cong \angle TUS \)
7. \( m\angle PUQ = m\angle TUS \)
8. \( m\angle PUQ + m\angle QUR = m\angle PUQ + m\angle SUR \)
9. \( m\angle QUR = m\angle SUR \)
10. \( \angle QUR \cong \angle SUR \)

\[ \therefore \text{Complements of congruent angles are congruent.} \]

**Example 10**
Prove that supplements of the same angle are congruent.

**Figure 2–9**

![Figure 2–9](image)
Study these examples carefully. Make sure that you understand the logic that leads from one statement to the next. Also, do you know why the listed definition or property justifies each statement? From these three proofs (Examples 8–10) we have our first three theorems.

- **THEOREM 2.1.** Congruent complements measure $45^\circ$.
- **THEOREM 2.2.** Complements of congruent angles are congruent.
- **THEOREM 2.3.** Supplements of the same angle are congruent.

So far, we have looked at examples of direct proofs. A **direct proof** uses deductive reasoning to show that the given information leads to the desired conclusion. It is also possible to prove theorems indirectly. An **indirect proof** shows that the negation for the desired conclusion leads to a contradiction. Since the negation of the conclusion is impossible, the conclusion must be true. A simple example of an indirect proof is when on a multiple choice test a student eliminates wrong choices until only the correct answer remains. On a matching section of an exam, sometimes I know the match for all items except for one. When I match this item with the remaining choice, I am using an indirect proof. Detectives often use indirect proofs by eliminating names from their list of possible suspects in order to find the guilty person. Our last example is an indirect proof. Notice that an indirect proof is most easily written as a paragraph.

**Example 11**

Prove that if two lines are parallel to the same line, then they are parallel to each other.

![Figure 2–11](image-url)

**Proof**

Suppose line $m$ is not parallel to line $n$. Then line $m$ and line $n$ intersect at some point, call it $R$. We know that $R$ is not on line $l$ because lines $m$ and $n$ are both parallel to line $l$. This means that there are two lines through point $R$ running parallel to line $l$. That would contradict postulate 5 described in the fourth section, Conjectures, Postulates, and Theorems, which states that there exists exactly one line, through a given point not on a line, that runs parallel to the given line. Therefore, $m$ and $n$ must be parallel to each other.

Hence, we have proven the following theorem.

- **THEOREM 2.4.** Two lines parallel to the same line are parallel to each other.
Let’s Check Your Mastery

Answer the following questions and then check your answers in the Answer Key at the back of this course manual. Do not submit your answers to Independent Study.

Complete the proof for Theorem 2.5 by giving the missing statements and reasons.

**THEOREM 2.5.** Congruent supplements measure 90°.

### Questions 18–20

**Proof**

*Given:* \( \angle A \cong \angle B \)

*Prove:* \( m\angle A = m\angle B = 90° \)

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reason</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( \angle A ) and ( \angle B ) are supplements.</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. ( m\angle A + m\angle B = 180° )</td>
<td>2.</td>
</tr>
<tr>
<td>3. ( \angle A \cong \angle B )</td>
<td>3. Given</td>
</tr>
<tr>
<td>4. ( m\angle A + m\angle A = 180° )</td>
<td>4. Definition of congruent angles</td>
</tr>
<tr>
<td>5. ( 2m\angle A = 180° )</td>
<td>5.</td>
</tr>
<tr>
<td>6. ( m\angle A = \frac{180°}{2} = 90° )</td>
<td>6. Distributive property</td>
</tr>
</tbody>
</table>

\[ \therefore m\angle A = m\angle B = 90°, \text{ so congruent supplements measure } 90°. \]

Complete the proof of Theorem 2.6 by giving the missing statements and reasons.

**THEOREM 2.6.** Complements of the same angle are congruent.

### Questions 21–23

**Proof**

*Given:* \( \angle 1 \) and \( \angle 3 \) are complements

*Prove:* \( \angle 1 \cong \angle 2 \)

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reason</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( \angle 1 ) and ( \angle 3 ) are complements</td>
<td>1.</td>
</tr>
<tr>
<td>2. ( \angle 2 ) and ( \angle 3 ) are complements.</td>
<td>2. Given</td>
</tr>
<tr>
<td>3. ( \angle 3 \cong \angle 3 )</td>
<td>3.</td>
</tr>
<tr>
<td>4. Complements of congruent angles are congruent (Theorem 2.2).</td>
<td>4.</td>
</tr>
</tbody>
</table>

\[ \therefore \text{ Complements of the same angle are congruent.} \]
Students usually find mathematical proofs strange and difficult to comprehend. If this is happening to you, don’t be discouraged. You will see more examples throughout this course. Just remember to study them carefully with the intent of understanding how early statements logically lead to the next statement, and why the mathematical reasons justify each statement. Very few people defend conclusions with the precision of a mathematical proof. However, the principles of deductive reasoning are important for making almost any decision.

It has already been mentioned that detectives must use deductive reasoning to catch criminals. Lawyers have the job of presenting the evidence in such a way that a judge and jury can follow the logic in the deductive reasoning, just like a proof. The defense lawyer tries to point out flaws in the reasoning that led to the prosecutor’s conclusion. If the definitions, postulates, and common notions used in law were as absolute as those in mathematics, then court decisions would be easily made. Either the logic is correct and the criminal is guilty or a counter example would exist proving one’s innocence. Even though life’s decisions aren’t as controlled as a mathematical proof, learning and following the principles of deductive reasoning will help one make better decisions.
Free tutors are available if you have questions about the lesson material. Use the contact information below to set up an appointment.
M–F from 7:00 a.m. to 7:00 p.m. (MST)
Phone: 1-800-914-8931
Email/IM: ismathtutor@byu.edu

Mark all answers here, then transfer them to your Speedback answer form. Drawing pictures may help you visualize the problems described. You may either submit your completed answer form to Independent Study for processing, or you may use WebGrade for immediate grading. See your Read Me First pamphlet for instructions.

Multiple Choice

For questions 1–3 determine if the conclusion is an example of inductive or deductive reasoning.

1. Cindy is waiting for a bus at a stop that services three routes: red, green, and blue. Cindy wants to take the red route bus. According to the bus schedule, the buses come in the order red, green, and then blue. When a blue route bus comes, Cindy knows that she should take the next bus to come.
   a. inductive reasoning
   b. deductive reasoning

2. To play in the children’s area at McDonald’s, a child must be shorter than 40 inches tall. A mother with three children knows that all three of her children can play at McDonald’s after checking to see that her tallest child is shorter than 40 inches.
   a. inductive reasoning
   b. deductive reasoning

3. During each of the last three mornings, Keith has bought a dozen chocolate doughnuts from the local bakery. When Keith comes into the bakery the next morning, the bakery clerk expects him to buy a dozen chocolate doughnuts.
   a. inductive reasoning
   b. deductive reasoning

For questions 4–6 determine if statement 2 is the inverse, converse, or contrapositive of statement 1.

4. Statement 1: If two lines intersect, then they form vertical angles.
   Statement 2: If two lines do not intersect, then they don’t form vertical angles.
   a. inverse
   b. converse
   c. contrapositive

5. Statement 1: If four points are collinear, then they are also coplanar.
Statement 2: If four points are noncoplanar, then they are also not collinear.
   a. inverse
   b. converse
   c. contrapositive

6. Statement 1: If two lines are perpendicular, then they form right angles.
   Statement 2: If two lines form right angles, then they are perpendicular.
   a. inverse
   b. converse
   c. contrapositive

For questions 7–11 choose the property that justifies each conclusion.

7. If \( AB \cong AD + DB \) and \( AB \cong AC + CB \), then \( AD + DB \equiv AC + CB \) because of the
   a. reflexive property
   b. symmetric property
   c. transitive property
   d. substitution property

8. If \( AD \equiv AC \), then \( AC \equiv AD \) because of the
   a. reflexive property
   b. symmetric property
   c. transitive property
   d. substitution property

9. If \( AB = AD + DB \) and \( AD = AC \), then \( AB = AC + DB \) because of the
   a. reflexive property
   b. symmetric property
   c. transitive property
   d. substitution property

10. We know that \( AC \equiv AC \) because of the
    a. reflexive property
    b. symmetric property
    c. transitive property
    d. substitution property

11. If \( BC = DC \), then \( BC + AC = DC + AC \) because of the
    a. symmetric property
    b. substitution property
    c. addition property
    d. distributive property

12. Which of the following are never used to justify statements made in the proof of a new theorem?
    a. definitions
    b. postulates
    c. conjecture
    d. previously proven theorems

**True/False**

13. Postulates are accepted as true without proof.
    a. true
    b. false

14. When using an indirect proof, we show that the negation of the desired conclusion leads to a contradiction.
    a. true
    b. false
Multiple Choice

Complete the proofs for Theorem 2.7 and Theorem 2.8 by selecting the correct statement or reason.

**THEOREM 2.7.** Supplements of congruent angles are congruent.

![Diagram of angles A, B, C, and D with supplements indicated]

<table>
<thead>
<tr>
<th>Questions 15–19</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Given:</strong> ( \angle A ) and ( \angle B ) are supplements. ( \angle C ) and ( \angle D ) are supplements. ( \angle A \cong \angle C )</td>
</tr>
<tr>
<td><strong>Prove:</strong> ( \angle B \cong \angle D )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reason</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( \angle A ) and ( \angle B ) are supplements.</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. ( m\angle A + m\angle B = 180^\circ )</td>
<td>2.</td>
</tr>
<tr>
<td>3. ( \angle C ) and ( \angle D ) are supplements.</td>
<td>3. Given</td>
</tr>
<tr>
<td>4. ( m\angle C + m\angle D = 180^\circ )</td>
<td>4.</td>
</tr>
<tr>
<td>5.</td>
<td>5. Transitive property</td>
</tr>
<tr>
<td>6. ( \angle A \cong \angle C )</td>
<td>6.</td>
</tr>
<tr>
<td>7. ( m\angle A = m\angle C )</td>
<td>7. Definition of congruent angles</td>
</tr>
<tr>
<td>8. ( m\angle A + m\angle B = m\angle A + m\angle D )</td>
<td>8. Substitution property</td>
</tr>
<tr>
<td>9.</td>
<td>9. Subtraction property</td>
</tr>
<tr>
<td>10. ( \angle B \cong \angle D )</td>
<td>10. Definition of congruent angles</td>
</tr>
</tbody>
</table>

\( \therefore \) Supplements of congruent angles are congruent.

15. Reason 2 of Theorem 2.7 should be
   a. definition of supplements
   b. definition of congruent angles
   c. addition property
   d. given

16. Reason 4 of Theorem 2.7 should be
   a. definition of supplements
   b. definition of congruent angles
   c. addition property
   d. given
17. Statement 5 of Theorem 2.7 should be
   a. \( \angle B \cong \angle D \)
   b. \( m \angle B = m \angle D \)
   c. \( m \angle A = m \angle C \)
   d. \( m \angle A + m \angle B = m \angle C + m \angle D \)

18. Reason 6 of Theorem 2.7 should be
   a. definition of supplements
   b. definition of congruent angles
   c. addition property
   d. given

19. Statement 9 of Theorem 2.7 should be
   a. \( \angle B \cong \angle D \)
   b. \( m \angle B = m \angle D \)
   c. \( m \angle A = m \angle C \)
   d. \( m \angle A + m \angle B = m \angle C + m \angle D \)

THEOREM 2.8. Vertical angles are congruent.

**Question 20**

![Diagram](image)

**Given:** \( \angle PTS \) and \( \angle QTR \) are vertical angles.

**Prove:** \( \angle PTS \cong \angle QTR \)

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reason</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( \angle PTS ) and ( \angle PTQ ) are supplements.</td>
<td>1. Definition of a linear pair of angles</td>
</tr>
<tr>
<td>2. ( \angle PTQ ) and ( \angle QTR ) are supplements.</td>
<td>2. Definition of a linear pair of angles</td>
</tr>
<tr>
<td>3.</td>
<td>3. Supplements of the same angle are congruent (Theorem 2.3).</td>
</tr>
</tbody>
</table>

\[ \therefore \text{Vertical angles are congruent.} \]

20. Statement 3 of Theorem 2.8 should be
   a. \( \angle PTQ \cong \angle PTQ \)
   b. \( \angle PTS \cong \angle QTR \)
   c. \( m \angle PTS + m \angle PTQ = 180^\circ \)
   d. \( m \angle PTQ + m \angle QTR = 180^\circ \)